Journal of Economic Dynamics & Control 181 (2025) 105205

Contents lists available at ScienceDirect

Journal of Economic Dynamics and Control

journal homepage: www.elsevier.com/locate/jedc

Unemployment and labor productivity comovement: the role of firm exit [⋆]

Miroslav Gabrovski [a], Mario Rafael Silva [b,*]

[a] Department of Economics, University of Hawai'i at Manoa, 2424 Maile Way Saunders Hall 516, 96822, Honolulu, HI, USA
[b] Department of Accountancy, Economics, and Finance, Hong Kong Baptist University, WLB 519 5 Renfrew Road, Kowloon Tong, Hong Kong

A R T I C L E I N F O

JEL classification:
E24
E32
J63
J64

Keywords:
Job destruction
Entry costs
Unemployment
Aggregate fluctuations
Dynamic correlations

A B S T R A C T

The Diamond-Mortensen-Pissarides model has been the primary workhorse for analyzing the dynamics of unemployment, vacancies, and market tightness over the business cycle. However, it predicts a near-perfect comovement between these variables and labor productivity, whereas the empirical correlation is only mild. We resolve this discrepancy by extending the model to incorporate sunk entry costs and finitely elastic vacancy creation, and by carefully distinguishing between business opportunity destruction and match separation as distinct sources of job loss. These features render vacancies a partially predetermined, positively valued stock variable. If the destruction rate is low, then most vacancies are inherited from the past and reflect historical rather than current productivity, breaking the tight unemployment-productivity link, while preserving strong correlations among labor market variables. We show that, when calibrated to information on job turnover and recall rates, the model reproduces the empirical contemporaneous and dynamic correlations between labor market variables and productivity while preserving the strong correlation between unemployment, vacancies, and the market tightness observed in the data.

1. Introduction

The Diamond-Mortensen-Pissarides model (DMP henceforth) has been the primary workhorse for studying the business cycle properties of unemployment, labor market tightness, and vacancies. Following Shimer (2005), much of the literature has focused on reproducing the empirically observed volatilities of these three labor market variables.[1] The majority of studies follow the tradition of the real business cycle literature and utilize technology shocks as the fundamental driving force behind business cycles. As a consequence, the model predicts near-perfect cross-correlation between productivity and the labor market variables. This result contrasts sharply with the data, in which the correlation is only mild, with coefficient of −0.43 (Fig. 1).[2]

Mortensen and Nagypal (2007) argue that this discrepancy in the implied correlation between model and data points to an important driving force of business cycle dynamics absent in the basic DMP framework. Following this line of thought,

⋆ We would like to thank Michael Choi, Ioannis Kospentaris, Victor Ortego-Marti, Guillaume Rocheteau, other participants of the UC Irvine Macroeconomics Workshop, and anonymous reviewers. We give a special acknowledgment to a reviewer for improving the interpretation of the model mechanism. Any remaining errors are our responsibility. Financial interests: none.

* Corresponding author.
E-mail addresses: mgabr@hawaii.edu (M. Gabrovski), msilva913@hkbu.edu.hk (M.R. Silva).

[1] Some notable examples include Hagedorn and Manovskii (2008), Hall and Milgrom (2008), Pissarides (2009), Ljungqvist and Sargent (2017).

[2] De-trending the series using the Hamilton regression filter instead with a forecast horizon of 2 years and using the 4 most available series, the correlation is −0.34.

https://doi.org/10.1016/j.jedc.2025.105205
Received 3 May 2025; Received in revised form 24 September 2025; Accepted 31 October 2025
Available online 3 November 2025
0165-1889/

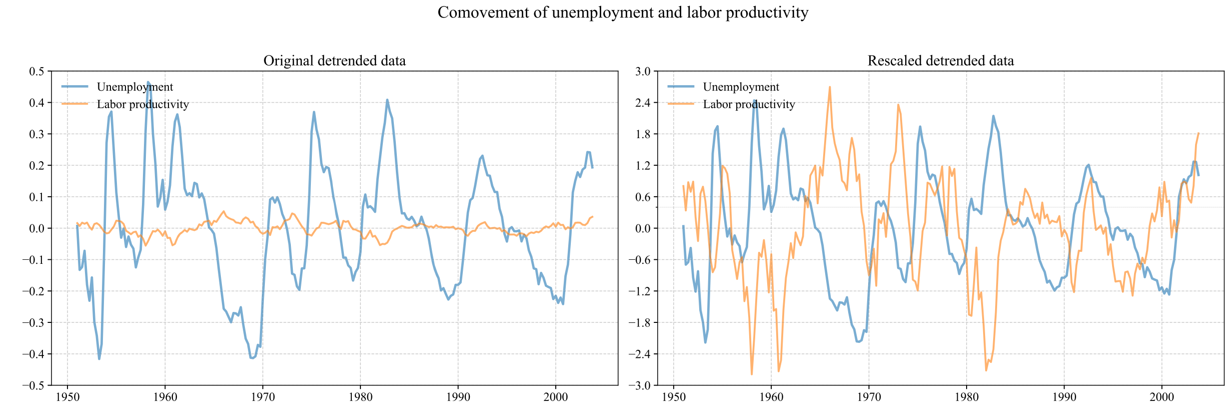

Fig. 1. Quarterly unemployment and labor productivity. Unemployment is measured in number of persons as Shimer (2005) and productivity is measured in terms of output per worker. The sample period is quarterly from 1951 to 2003. Each series is logged and detrended using an HP filter with smoothing parameter of 10^5. In the right panel, each series is also rescaled to have a standard deviation of unity.

Barnichon (2010) develops a model with both technology and demand shocks that can reproduce a mildly counter-cyclical unemployment rate. Similarly, Gervais et al. (2015) incorporate a proficiency ladder in the DMP model and show that shocks to the ease of learning-by-doing can generate an unemployment-productivity correlation close to that the data. Coles and Moghaddasi Kelishomi (2018) build a model with both technology and separation shocks that breaks the near-perfect correlation between productivity and the labor market variables.

In contrast to this existing literature, we propose a mechanism which endogenously reduces the magnitude of the correlation between the labor market variables and productivity, and relies only on a single technology shock as the underlying source of volatility in the economy. Specifically, we augment the DMP model with sunk vacancy creation costs and carefully distinguish between firm-worker separations and job/product line destruction, following which the vacancy is lost alongside the job. A *separation* shock dissolves the match, but lets the firm repost its vacancy without incurring any additional costs, whereas a *destruction/obsolescence* shock destroys both the firm-worker match and the associated vacancy leading the firm to completely exit the labor market. As a consequence, there are three types of vacancies in our economy: i) newly created vacancies from entrants; ii) surviving vacancies from past periods that did not suffer an obsolescence shock, but were not able to find a worker either; iii) reposted vacancies associated with jobs which suffered a separation shock, but not a destruction shock.

When the destruction shock occurs less frequently, most vacancies in the pool are either surviving or reposted. Consequently, vacancies — and by extension the market tightness and unemployment — tend to comove more closely with past productivity than with the current state of technology, because the decision to *initially* create those vacancies was taken in the past. This results in unemployment that is only mildly correlated with productivity. Our analysis shows that carefully calibrating the model using data on job separations, layoffs, and recalls from the Job Openings and Labor Turnover Survey (JOLTS) implies a value for the destruction shock that can indeed reconcile the model-implied cross-correlations with those observed in the data.

Following Diamond (1982), a small but growing literature highlights the importance of sunk vacancy creation costs in the context of the DMP model for explaining labor market dynamics.[3] We contribute to this literature by highlighting its importance for the comovement between labor productivity and labor market variables (unemployment, vacancies, market tightness). Our model environment nests (Coles and Moghaddasi Kelishomi, 2018) as a special case in which all worker-firm pair dissolutions are due to job destruction, i.e. there are no (voluntary) separations.[4] Specifically, creating a new vacancy requires an up-front investment in a new technology. This sunk investment is drawn randomly from a known exogenous distribution. Consequently, each vacancy carries a positive asset value and, as a result, when a firm and worker separate voluntarily, the firm strictly prefers to keep its vacancy open.

[3] Notable examples include Fujita and Ramey (2007), who focus on the sluggish response of the market tightness to productivity shocks; Shao and Silos (2013), who stress the dynamics of the value of a vacancy; Coles and Moghaddasi Kelishomi (2018), who emphasize the drivers of unemployment volatility over the cycle; Mercan and Schoefer (2020), who concentrate on replacement hiring; Haefke and Reiter (2020), who examine match cyclicality and wages; Potter (2024), who focuses on modern search technologies, and Qiu (2023), who analyzes vacancy dynamics and the decision of whether or not to participate in the labor force. The environment in Coles and Moghaddasi Kelishomi (2018) is a special case of ours in which the separation rate is zero. Our environment also nests that of Fujita and Ramey (2007) as a special case, since we use a more flexible sunk investment cost function. Compared to the model in our paper, the one by Shao and Silos (2013) differs mostly from the role of capital in their economy as the driving source of congestion in vacancy creation — the more firms enter, the higher the demand for capital which increases its rental price and, as a consequence, the equilibrium cost of vacancy creation. The model in Mercan and Schoefer (2020) incorporates job-to-job flows, which we abstract from.

[4] We refer to voluntary separations as those initiated by either the firm or the worker for reasons unrelated to obsolescence or the loss of the underlying business opportunity. Such separations are the result of quits, firing for cause, retirement, etc. and allow the firm to repost its vacancy upon separation with the worker. A destruction shock instead destroys both the match and associated vacancy.

We distinguish this separation shock from a destruction shock, in which the vacancy is lost alongside the job. Such a disturbance makes the firm's product obsolete, or equivalently destroys its business opportunity. Thus, when a destruction shock hits, the firm exits the labor market altogether.

A negative productivity shock lowers future expected profits, which dampens vacancy creation. In the standard DMP environment, vacancies fall instantaneously, which causes labor market variables to comove almost perfectly with productivity. In our setting, on the other hand, vacancies are long-lived assets with a positive value — a stock variable. Specifically, we distinguish between three types of vacancies: i) newly created vacancies; ii) surviving vacancies that were originally created in previous periods but failed to match with a worker; iii) reposted vacancies by firms who lost their worker, but not their business opportunity. Hence, the number of vacancies in the market is correlated not only with the current technology shock, but also with past ones, since the decision to *originally* open the surviving and reposted vacancies was taken in the past. As the expected life of a job (whether filled or vacant) rises, so does the history of past shocks that affects the pool of vacancies today. This entails a larger fraction of vacancies created in the past and thus a lower correlation between vacancies and current productivity. Consequently, the magnitude of the cross-correlations between labor market variables and productivity in the model depends on how long-lived business opportunities are, i.e. the size of the destruction shock.

Consequently, disciplining the size of the destruction shock plays a central role. Initial vacancy creation requires an up-front sunk investment on the part of the firm in order to capture a business opportunity. As the destruction shock most closely resembles the loss of a business opportunity, it is appropriate to calibrate it using data on job loss due to firm exit and product obsolescence. Unfortunately, there are no such direct aggregate estimates. The closest series available is the "layoffs and discharges" reported by JOLTS, which is about 1.4 % monthly. However, this number includes temporary layoffs as well as firing for cause — events consistent with the separation shock in our model. To overcome this issue we use the findings from Fujita and Moscarini (2017) and Lam and Qiu (2022) who report that about 30 % (40 %) of all separations resulted in later recalls of the worker back to the job. Thus, we should expect only 70 % (60 %) of the layoffs and discharges to be permanent. This implies an annual destruction rate of about 11.5 % (9.84 %). As it turns out, this number is consistent with the destruction rate implied by the data on job separations from the Business Dynamics and Statistics (BDS) database and with micro-level estimates on firm exit. Thus, our benchmark calibration targets an annual destruction rate of 10 %. This implies a destruction shock of about 0.9 % monthly. [5]

Under this baseline calibration our model reproduces the mild correlation between productivity and all three labor market variables of interest (vacancies, the market tightness, and unemployment) well. Moreover, the cross-correlations between the labor market variables themselves remain strong. We also highlight how our strategy of calibrating the destruction rate complements and improves on several shortcomings in the existing literature in Section 3.1. In addition, we demonstrate that matching the empirical correlation between unemployment and productivity does not necessarily compromise the model's ability to replicate the relative volatility of unemployment with respect to productivity. Specifically, increasing the value of the elasticity of entry with respect to vacancies from its benchmark level significantly amplifies unemployment volatility, while only moderately affecting the magnitude of the correlation.

Related literature. Although the literature has mainly focused on the unemployment volatility puzzle, several studies have also stressed the correlation between labor productivity and unemployment. Notably, Barnichon (2010) highlights the stylized fact that these two series are only mildly correlated, using a variety of productivity measures. Furthermore, he finds the cross-correlation to be negative pre-1984 and positive post-1984, though it is mild in both periods. Barnichon (2010) explains the empirically observed sign change of the correlation using demand and supply shocks and nominal rigidities. In related work (Hagedorn and Manovskii, 2011) examine some of the empirical shortcomings of the DMP model, including the discrepancy between the theoretically predicted cross-correlation between unemployment and productivity and its empirical counterpart. The authors reproduce the mild magnitude of the correlation by incorporating a stochastic home production value and a time-to-build lag for vacancies. Intuitively, both features reduce the strength of the correlation between productivity and unemployment almost mechanically. First, wages, and consequently the firm's surplus from the match, depend on home production, so the model features a second independent source of volatility that affects firms' vacancy posting decisions. Second, because of the time-to-build lag, vacancies which enter the labor market today are not correlated with current productivity shocks. Furthermore, the time-to-build assumption allows the authors to match a qualitative feature of the dynamic correlations in the data — the peak correlation between vacancies and productivity occurs when productivity is lagged two quarters.

Gervais et al. (2015) develop a model with learning-by-doing that is able to reproduce the empirically observed cross-correlation using a single source of exogenous uncertainty. They examine shocks to the rate with which workers learn on the job in lieu of technology disturbances. Given a positive learning shock, firms increase hiring because they expect high future profits, which reduces unemployment. The effect of the shock on productivity, however, is indirect and only works through a labor force composition effect. A higher rate of learning makes it easier for workers to increase their proficiency, which raises aggregate productivity subject to a lag. This breaks the immediate response of productivity in the standard DMP model and delivers a lower magnitude correlation between productivity and unemployment.

In contrast to the existing literature, our model features a productivity shock as the single source of exogenous volatility in the model. We believe our mechanism is worthy of investigation because it allows us to more fully capture the response of unemployment to technology shocks. This is important for two reasons. First, even though unemployment dynamics are likely to be affected by

[5] For a detailed discussion see Section 3.1 and the robustness exercises therein. Of course, both in our model and in the data there is job loss due to reasons unrelated to obsolescence and firm exit. So we calibrate our separation shock to be consistent with a *total* firm-worker separation rate of 3.4 % monthly, following the evidence in JOLTS.

other exogenous shocks such as monetary shocks and demand shocks (Christiano et al., 2016), these shocks imply different impulse response functions (IRFs) of unemployment. Understanding under what conditions the DMP class of models can match the empirical IRFs is of interest. Second, understanding the dynamics of unemployment following a technology shock has important implications when 1 aims to decompose the relative importance of different shocks to the dynamics of unemployment. For example, Mortensen and Nagypal (2007) conclude that shocks other than technology must be important for unemployment, citing the mild correlation observed in the data. Our model implies that this is not necessarily the case. More importantly, any model which implies a perfect correlation between unemployment and labor productivity is bound to underestimate the importance of productivity shocks when several exogenous sources of volatility are considered.

2. Model

2.1. Environment

Our environment closely follows a conventional equilibrium unemployment model in discrete time, e.g. Pissarides (2000). The only point of departure is in the vacancy entry mechanism — we assume sunk vacancy creation costs alongside a finitely elastic vacancy creation, i.e. there is congestion in vacancy creation. Unemployed workers search for jobs, firms search for workers to fill their vacancies, and matches are formed according to a matching function. Once matched, workers and firms decide on wages using Nash bargaining and the match persists until the pair exogenously separates. There is a unit measure of firms that can create vacancies. In each period firms receive access to a new independent business opportunity, which can be undertaken by paying an investment cost x. This cost reflects, for example, the costs associated with R&D and taking a new product to the production phase, or the costs associated with purchasing capital. Let Q_t denote the value of posting a vacancy at time t, and suppose each firm draws an investment cost from a known distribution G. Given that firms undertake their business opportunity if and only if $x \leq Q_t$, the aggregate amount of new vacancy creation is $e_t = G(Q_t)$.[6]

Each period a number $M(u_t, v_t)$ of firm-worker pairs are formed, where u_t denotes unemployment and v_t the number of vacancies. As is standard in the literature, the matching function is increasing and concave in each of its arguments and exhibits constant returns to scale. Thus, the job-filling rate for firms is $q(\theta_t) \equiv M(u_t, v_t)/v_t = M(\theta_t^{-1}, 1)$ and the job-finding rate is $f(\theta_t) \equiv M(u_t, v_t)/u_t = M(1, \theta_t)$, where $\theta_t \equiv v_t/u_t$ denotes the market tightness. Firms and workers separate due to one of two reasons. First, with probability s the firm-worker pair chooses to dissolve the match. We call this a separation shock and it is meant to capture match dissolution due to quits, firing for cause, retirement, or similar. Importantly, following a separation shock, the firm keeps its business opportunity and any job-specific capital it may have purchased. Second, with probability δ the firm's business opportunity becomes obsolete, i.e. the job is destroyed. In that event the firm loses the job as well as the opportunity to repost its vacancy. We call this a destruction/obsolescence shock and it is meant to capture firm exit, downsizing, permanent layoffs, or similar. Thus, in our economy the *total* separation rate is $\tau \equiv 1 - (1-\delta)(1-s) \approx s + \delta$.[7]

Fig. 2 summarizes the timing in our model. At the beginning of each period the aggregate productivity shock is realized and agents observe the current level of productivity p_t. Next, firms receive their investment opportunities and make entry decisions. Third, production takes place: firm-worker pairs that are matched produce p_t, workers are paid a wage w_t, and unemployed workers receive benefits b_t. Furthermore, at this stage firms which have an unfilled vacancy pay the vacancy posting cost γ. Fourth, matching takes place. Fifth, worker separation and firm destruction take place. A match formed in period t is not subject to a separation shock at time t, but may be hit with a destruction shock. Moreover, newly created vacancies can also be destroyed.

2.2. Bellman equations

The value of a vacancy, Q_t, comprises several terms. First, firms must pay a vacancy posting cost γ in order to search for workers in the labor market. If they are matched with a worker, which occurs with probability $q(\theta_t)$, their vacancy transitions to a filled job. Otherwise, they keep the opportunity to search for a worker next period. Finally, firms discount the future with a factor β and expect their business opportunity to remain viable with probability $(1-\delta)$. Letting J_t denote the value of a filled job, the value of a vacancy Q_t satisfies

$$Q_t = -\gamma + \beta(1-\delta)\mathbb{E}_t\left[q(\theta_t)J_{t+1} + (1-q(\theta_t))Q_{t+1}\right]. \tag{1}$$

Let $K_t \equiv Q_t - \beta(1-\delta)\mathbb{E}_t Q_{t+1}$ denote the flow value of a vacancy. Then (1) can be rearranged as

$$\frac{\gamma + K_t}{q(\theta_t)} = \beta(1-\delta)\mathbb{E}_t[J_{t+1} - Q_{t+1}]. \tag{2}$$

Eq. (2) shows that the discounted expected surplus next period equals the average hiring cost, appropriately adjusted for the sunk entry cost.

6 Appendix C shows that the model is isomorphic to having a constant sunk entry cost that depends positively on the number of entrants. Our entry condition implies there is congestion in vacancy creation — the more firms post vacancies each period, the higher the average investment firms must make. There is an analogous congestion mechanism if instead firms compete for new business opportunities in the form of innovations. See, for example, Gabrovski (2019) and Gabrovski (2022).

7 Our environment thus nests (Coles and Moghaddasi Kelishomi, 2018), who assume all job loss is due to destruction, i.e. $s = 0$.

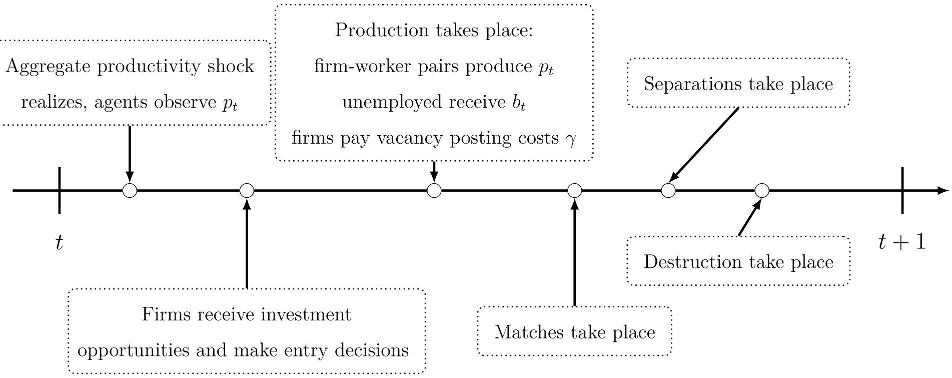

Fig. 2. Labor market timing.

A filled job has productivity p_t. Firms pay workers a wage w_t, so the per-period profits are $p_t - w_t$. With probability $(1-s)(1-\delta)$ the firm and the worker do not separate and the business opportunity does not become obsolete, so the firm keeps its filled job next period. There is a chance $s(1-\delta)$ that the firm-worker pair dissolves due to a voluntary separation (rather than destruction). In that event, the firm keeps a vacancy with expected value $\mathbb{E}_t Q_{t+1}$ and can search for a new worker next period. Lastly, in the event that the business opportunity is destroyed, the firm-worker pair dissolves and the firm exits the market. Thus,

$$\begin{aligned} J_t &= p_t - w_t + \beta\left[(1-s)(1-\delta)\mathbb{E}_t J_{t+1} + s(1-\delta)\mathbb{E}_t Q_{t+1}\right] \\ &= p_t - w_t + \beta(1-\delta)\mathbb{E}_t[(1-s)(J_{t+1} - Q_{t+1}) + Q_{t+1}]. \end{aligned} \tag{3}$$

An unemployed worker receives benefits b. With probability $f(\theta_t)$ she is matched with a firm and, conditional on that, there is a $(1-\delta)$ chance the job survives until the beginning of next period. In that event the worker becomes employed. Otherwise, the worker remains unemployed. Thus, the value of unemployment, U_t, satisfies

$$U_t = b + \beta\left[(1-\delta)f(\theta_t)\mathbb{E}_t W_{t+1} + [1-(1-\delta)f(\theta_t)]\mathbb{E}_t U_{t+1}\right]. \tag{4}$$

An employed worker receives wages w_t. She keeps the job whenever the firm-worker pair does not separate and the firm survives the destruction shock, i.e. with probability $(1-\delta)(1-s)$. Otherwise, the worker loses the job and transitions to unemployment next period. The value of the job for the worker, W_t, satisfies

$$W_t = w_t + \beta\left[(1-\tau)\mathbb{E}_t W_{t+1} + \tau\mathbb{E}_t U_{t+1}\right]. \tag{5}$$

2.3. Wages, laws of motion, and entry

As is standard in the literature, wages are determined according to Nash bargaining:

$$w_t = arg, max_{w_t} [J_t - Q_t]^{1-\alpha}[W_t - U_t]^{\alpha},$$

where α is the worker's bargaining power. The first-order condition implies that each party receives a fixed fraction of the total surplus:

$$\alpha(J_t - Q_t) = (1-\alpha)(W_t - U_t). \tag{6}$$

Using the surplus sharing rule (6) together with the four Bellman Eqs. (1), (3)–(5) yields an expression of the wage that generalizes the one in the DMP model:

$$w_t = \alpha\left[p_t - K_t + \theta_t(\gamma + K_t)\right] + (1-\alpha)b. \tag{7}$$

The wage is a weighted average between the flow payoff from the match and the worker's outside option of receiving unemployment benefits. The benefit of the match constitutes the output together with the search costs saved from not posting a vacancy, $p_t + \theta_t(\gamma + K_t)$.

The only difference between (7) and the corresponding equation in the workhorse DMP model is that the vacancy costs include both γ and the flow entry cost K_t. Additionally, vacancies are an asset with a positive value and are subject to congestion in our economy. Consequently, the wage includes an additional term: $-K_t$. This term reflects the fact that the outside option of firms in the bargaining game has a flow value K_t. If vacancies are expected to be more valuable tomorrow, then the firm will have an easier time recruiting a worker tomorrow, which raises the attractiveness of firms' outside option and subsequently reduces the wage. Alternatively, if the expected value of a vacancy tomorrow is lower the firm is more eager to match with a worker today and is thus willing to offer a higher wage.

Finally, we close the model by specifying the laws of motion for vacancies and unemployment, as well as the entry decisions of firms. Because firms must incur a sunk investment cost to enter the labor market, vacancies are partially predetermined. The vacancy pool comprises three components: (i) *newly created* vacancies e_t, posted by firms entering the labor market this period; (ii) *surviving* vacancies, $(1-\delta)q(\theta_{t-1})v_{t-1}$, which are unmatched vacancies from the previous period that avoided destruction; and (iii) *reposted* vacancies, $(1-\delta)s(1-u_{t-1})$, which arise when firm-worker matches separate, but the job opportunity is not destroyed. Thus, the law of motion for vacancies satisfies

$$v_t = (1-\delta)[1-q(\theta_{t-1})]v_{t-1} + (1-\delta)s(1-u_{t-1}) + e_t. \tag{8}$$

The law of motion for unemployment also departs from the standard DMP model due to the presence of the destruction shock. Specifically, worker-firm matches are exposed to a destruction shock immediately after being formed, before any production occurs. As a result, only a fraction $(1-\delta)f(\theta_{t-1})$ of unemployed workers who are matched in the previous period transition into employment in the current period. The rest either remain unemployed or exit employment due to separations. Thus, unemployment evolves according to

$$u_t = [1-(1-\delta)f(\theta_{t-1})]u_{t-1} + \tau(1-u_{t-1}). \tag{9}$$

Free entry implies that new vacancies are posted until the sunk entry cost equals the value of a vacancy: $e_t = G(Q_t)$. We follow Broer et al. (2025) and use a bounded power law distribution for $G(\cdot)$:[8]

$$G(x) = (x/x_m)^{\xi}, \quad x \in [0, x_m].$$

Provided that the value of the vacancy Q_t is less than x_m we can invert the CDF to find $Q_t = (e_t)^{1/\xi}x_m$.[9] Thus, the elasticity of entrants to the value of a vacancy, ξ, is positive and finite. In contrast, under the DMP model with no congestion this elasticity is infinite.

2.4. The job creation condition

Combining the free entry condition, $Q_t = e_t^{1/\xi}x_m$, and the Bellman equations for vacancies (1) and for filled jobs (3) yields the job creation condition in our economy:

$$\frac{\gamma + K_t}{q(\theta_t)} = \beta(1-\delta)\mathbb{E}_t\left[p_{t+1} - w_{t+1} - K_{t+1} + (1-s)\left(\frac{\gamma + K_{t+1}}{q(\theta_{t+1})}\right)\right]. \tag{10}$$

The interpretation of (10) is standard: the left-hand side of the equation represents the expected costs of posting and maintaining a vacancy, whereas the right-hand side is the expected profit. The job creation costs in our environment account for the role of congestion. For example, if many firms are entering the market today then K_t rises and an entrant may choose to delay entry. This smoothing mechanism yields a hump-shaped response in vacancies, a feature consistent with the empirical evidence provided by Fujita and Ramey (2007). The expected benefit from posting a vacancy is the expected discounted profits next period, $p_{t+1} - w_{t+1}$, plus the continuation value of the vacancy in the event the firm and worker separate, $(1-s)[(\gamma + K_{t+1})/q(\theta_{t+1})]$, net of the expected flow costs, K_{t+1}.

2.5. The exogenous productivity process

A single technology shock is the sole source of volatility in our economy. We follow much of the existing literature and impose that the natural logarithm of productivity follows an AR(1) process:

$$\log(p_t) = (1-\rho)\log\overline{p} + \rho\log(p_{t-1}) + \epsilon_t, \tag{11}$$

where ρ is a persistence parameter and $\epsilon_t \sim N(0,\sigma)$ is white noise.

2.6. Equilibrium definition, existence and uniqueness

We now have the necessary ingredients to define equilibrium:

Definition 1. An equilibrium is an infinite, bounded sequence of productivity, wages, market tightness, entrants, vacancies, and unemployment $\{p_t, w_t, \theta_t, e_t, v_t, u_t\}_{t=0}^{\infty}$ such that, given initial conditions (u_0, v_0, p_0), (i) firms set entry optimally according to (10); (ii) the wage solves the Nash Bargaining problem between the firm and the worker as in (7); (iii) vacancies follow the low of motion (8); (iv) unemployment follows the law of motion (9); and (v) productivity follows the AR(1) process defined in (11).

Defining the rate of time preference as $r = 1/\beta - 1$ allows us to also summarize the steady state equilibrium in the economy:

Definition 2. A steady-state equilibrium is a list $(Q, K, e, w, \theta, u, v)$ such that

$$\frac{\gamma + K}{q(\theta)} = \frac{1-\delta}{r+\tau}(p - w - K) \tag{12}$$

$$Q = e^{1/\xi}x_m$$

[8] This generalizes the upper bound respect to Coles and Moghaddasi Kelishomi (2018), who set it equal to one. Beaudry et al. (2018) and Potter (2024) use similar functional forms.

[9] Note that $Q_t < x_m$ whenever $e_t < 1$, a condition which is trivially satisfied in our quantitative exercises.

$$K = Q\frac{r+\delta}{1+r} \tag{13}$$

$$w = \alpha[p - K + \theta(\gamma + K)] + (1-\alpha)b \tag{14}$$

$$u = \frac{\tau}{\tau + (1-\delta)f(\theta)} \tag{15}$$

$$e = \delta(v + 1 - u) \tag{16}$$

$$\theta = \frac{v}{u}$$

The economy generally at most one steady state, which exists under mild conditions:

Proposition 1. *There is at most one steady state. The steady state exists if and only if*

$$\gamma < \frac{(1-\delta)(1-\alpha)(p-b)}{r+\tau}$$

A proof is included in Appendix A.3.2.

3. Quantitative analysis

We now present the main results, demonstrating that the model can reproduce the mild correlation between unemployment and labor productivity observed in the data. We argue that the mild cross-correlation arises for two key reasons. First, vacancies are a predetermined variable that evolve according to the law of motion (8). When the destruction shock δ is relatively low, the contribution of entrants to the stock of vacancies is limited, resulting in a weaker correlation between vacancies and labor productivity. Second, the model incorporates a congestion channel: the average cost of entry increases with the number of entrants.

Section 3.1 details the calibration strategy, documents the steady-state shares, and plots the steady-state distributions of vacancies by age and total time filled.[10] Section 3.2 provides histograms of quarterly averages for unemployment, vacancies, and market tightness, and illustrates how the model's nonlinear mechanisms affect both the means and skewness of these distributions. Section 3.3 presents impulse responses for different values of the destruction rate δ, revealing its effect on relevant correlations. Additionally, we find that an increase in the elasticity parameter ξ leads to greater unemployment volatility, without significantly altering the mild unemployment-productivity correlation. Finally, Section 3.4 shows that the model provides a good fit for the dynamic correlations of productivity with both unemployment and vacancies.

3.1. Calibration and steady-state shares

In our framework, creating a vacancy is intrinsically linked to a business opportunity. Job loss in the economy results from two distinct processes: (i) a separation between firm and worker that dissolves the match but preserves the firm's business opportunity, and (ii) destruction of the business opportunity that renders the firm's product no longer viable, forcing the firm to exit the labor market. The first source of job loss aligns with the standard DMP model's view of separations: matches dissolve when workers relocate, change jobs due to management conflicts, are terminated for cause, etc. The second source relates to the firm's business environment: matches end when competitors capture market share, products become obsolete, or firms cease operations. Therefore, we pay special attention to carefully distinguish between these two sources of separation in our calibration.

To begin with, we calibrate the economy at a monthly frequency. The total separation rate is set to $\tau = 3.4\,\%$, which can be imputed using unemployment flows as Shimer (2005) and Coles and Moghaddasi Kelishomi (2018) and is consistent with the aggregate separation rate reported in JOLTS. To disentangle the contribution of the δ and s shocks to the total rate, we delve further into the JOLTS data which reports an average "quits" rate of about $2\,\%$ and a rate for "layoffs and discharges" of about $1.4\,\%$.[11] The survey defines quits as a voluntary separation where the worker leaves on her own volition, whereas layoffs and discharges encompasses separations initiated by the employer. Through the lens of our model, quits should be interpreted as an s-type shock, but it is not clear how layoffs and discharges are to be interpreted, since they include, for example, firing for cause and temporary layoffs. Through the lens of our model, separations such as firing for cause and temporary layoffs should be counted as an s-type shock because they do not affect the firm's business opportunity and do not preclude it from reposting its vacancy. Permanent job eliminations due to downsizing, firm exit, or permanent layoffs should, on the other hand, be counted as a δ-type shock because they most likely are associated with the destruction of the underlying business opportunity.

To disentangle the relative importance of these two types of shocks in the series on layoffs and discharges we turn to the evidence in Fujita and Moscarini (2017). The authors report that around $30\,\%$ of all firm-worker separations result in a recall: the worker is rehired at the firm. Applying this estimate to our data yields a permanent (excluding recall) layoff rate of around $0.98\,\%$ monthly.

[10] Notably, lower values of δ shift both distributions toward much longer horizons. We are very thankful to an anononymous reviewer for encouraging us to do this exercise.

[11] JOLTS also reports a $0.2\,\%$ rate of separations due to reasons outside the control of either the firm or the worker, such as retirement, death, or disability. Through the lens of our model these separations are the result of an s-type shock.

Similarly, Lam and Qiu (2022) estimate a recall rate around 40 % to 60 %.[12] These estimates yield a permanent layoff rate of about 0.56 % — 0.84 %. Overall, this suggests a conservative value of the destruction shock is $\delta = 0.009$ (or 10 % annually) and a separation shock $s = 0.026$.

Alternatively, one can turn to data on product obsolescence and firm downsizing/exit to calibrate δ, since the destruction shock in our model is associated with the loss of the firm's business opportunity. This is especially true given the results in Bernard et al. (2010) and Lee and Mukoyama (2015) who argue that product turnover and firm exit are both linked to employment turnover. Using micro-level evidence (Broda and Weinstein, 2010) find an annual product destruction rate of 3 % — a value consistent with the 3 % obsolescence rate (Comin and Gertler, 2006) calibrate using balanced growth path restrictions from the U.S. data. These values are also within the ball park of the 5 % — 6 % annual destruction rate implied by the estimates in Bernard et al. (2010). Alternatively, Broda and Weinstein (2010) and Lee and Mukoyama (2015) both report firm exit rates of about 10 % annually. Of course, these numbers do not represent a perfect estimate of the δ shock in our model. On the one hand, exiting and shrinking firms tend to be smaller, so they might have a smaller contribution to total separations in the economy. On the other hand, there may be δ-type destruction shocks that occur in firms that are neither exiting nor closing down production lines. Nonetheless, the estimates appear broadly consistent with our calculation of δ using the data from JOLTS. One approach to partially address the aforementioned issues is to look at the data on job separations from the Business Dynamics Statistics (BDS) database. The BDS reports the rate of all separations that occur in exiting and shrinking establishments to be around 14 % annually. This number is likely to be an over-estimation since it includes all types of separations, including voluntary quits. The BDS also reports a job loss rate of 5 % coming from exiting establishments. This is likely to be an under-estimation of our δ shock since it does not include the job destruction which takes place in establishments that do not exit. Thus, the true value of δ would lie somewhere in the middle. Overall, we believe the aggregate evidence on layoffs from JOLTS, job separations from BDS, and the micro-level evidence on firm exit and product obsolescence from the existing literature points towards a value of the destruction shock around 10 % annually. However, we also explore the robustness of our results and show that for a broad range of values of δ consistent with the aforementioned evidence, our model delivers a mild correlation between unemployment and labor productivity close to that observed in the data.

We should highlight that existing studies within the labor search literature have targeted job destruction rates in alternative ways. For example, Coles and Moghaddasi Kelishomi (2018) set the separation rate to zero and attribute all job separations in the data to destruction. Fujita and Ramey (2007), on the other hand, identify the destruction and separation rates using evidence on total job losses from the Business Employment Dynamics (BED) program, coupled with the usual moments of the job-finding rate and steady state unemployment. A similar strategy is used by Shao and Silos (2013) who pin down the destruction rate using evidence on total job losses from Shimer (2005) and a moment restriction for the steady state level of unemployment. Mercan and Schoefer (2020) use survey evidence from German employers to distinguish between hiring aimed at replacing workers who have quit (replacement hiring) and new job creation to separately calibrate their destruction and separation rates.

Compared to Coles and Moghaddasi Kelishomi (2018), our calibration strategy leverages evidence on layoffs and and recalls, eliminating the need to attribute all job separations to destruction.[13] Unlike Mercan and Schoefer (2020), our approach relies exclusively on U.S. data. While they accurately derive the separation rate from data on new jobs versus re-hires, their German data presents a limitation for our purposes, as German and U.S. labor markets likely differ in key aspects affecting destruction and separation rates and their business cycle properties. Furthermore, we utilize only moments readily available from public data. Fujita and Ramey (2007) use job destruction evidence from the BED program reported in Faberman et al. (2004) for their calibration. Since job destruction is defined as "the gross number of jobs lost at establishments either closing down or contracting their workforce" (Faberman, 2004, p.1), the estimate of the destruction rate in their calibration likely over-estimates δ because there are voluntary separations, i.e. s-type shocks, in shrinking establishments as well.

Following Den Haan et al. (2000), the matching function is set to $M(u,v) = uv/(u^{\nu_L} + v^{\nu_L})^{1/\nu_L}$, which bounds matching probabilities between 0 and 1:

$$f(\theta) = \frac{\theta}{(1+\theta^{\nu_L})^{1/\nu_L}}, \quad q(\theta) = \frac{1}{(1+\theta^{\nu_L})^{1/\nu_L}}.$$

As in Coles and Moghaddasi Kelishomi (2018), we set the average duration of unemployment $1/[(1-\delta)f(\theta)] = 2.2$ months, so that $(1-\delta)f(\theta) = 45\,\%$. We also set the average vacancy duration to 3 weeks (Blanchard and Summers, 1986), which implies $(1-\delta)q(\theta) = 1-(1-1/3)^4$. Thus, $q(\theta) = 0.8/(1-\delta)$. Together these two moments yield $\nu_L = 1.58$.

The discount factor $\beta = 0.9967$ is set to match an annual discount rate of 4 %. Further, we set the vacancy posting costs γ to zero, following Coles and Moghaddasi Kelishomi (2018) and set $b = 0.9$, which is between the value 0.71 and 0.95 imputed by Hall and Milgrom (2008) and Hagedorn and Manovskii (2008), respectively. The elasticity of entry ξ is set to 1, so that the distribution $G(Q_t)$ is uniform on $(0, x_m)$. We set the bargaining power to target an approximate steady-state elasticity of wages to technology of 0.6.[14] The level parameter p just determines units, so we set the value to satisfy a steady-state wage equal to 1. The upper bound x_m is indirectly pinned down by the vacancy filling rate via the number of new vacancies e. The only source of uncertainty in our model is

[12] Lam and Qiu (2022) do not report aggregate recall rates, but rather they report recall rates by age groups. The authors estimate a recall rate for all workers between 30 % and 60 % and a recall rate for laid off workers, in particular, between 40 % and 80 %. Thus, our target of a 40 % recall rate is a conservative one.

[13] To be precise, Coles and Moghaddasi Kelishomi (2018) specify their model so that all separations lead to the loss of a vacancy. Thus, in contrast to their framework we make the distinction between separations and destruction.

[14] Specifically, we find α such that $\alpha p/w = 0.6$, which abstracts from feedback from productivity to the flow entry cost K.

Table 1
Calibration targets alongside calibrated parameter values and their symbolic representation.

Targets	Value	Parameter	Calibration
Real interest rate (annual)	0.04	r	0.003
Business destruction rate (annual)	0.10	δ	0.009
Replacement ratio b/p	0.9	b	0.9
Elasticity of vacancy value	1.0	ξ^{-1}	1.0
Steady-state productivity	1.0	p	1.0
Aggregate separation rate (monthly)	0.034	s	0.026
Wage elasticity to productivity	0.6	α	0.566
Job finding rate (monthly)	0.45	v_L	1.58
Vacancy filling rate (monthly)	0.8	x_m	535.0

Table 2
Steady-state shares given calibration.

Share	Symbol	Value
Vacancy rate	v	0.0395
Unemployment rate	u	0.0702
Market tightness	θ	0.563
New vacancy share	e/v	0.214
Surviving vacancy share	$(1-\delta)(1-q(\theta))$	0.191
Reposted vacancy share	$(1-\delta)s(1-u)/v$	0.594
Average vacancy age since birth (years)	–	7.1
Average vacancy time filled (years)	–	6.79
Value of a vacancy	Q	4.54
Value of a filled job	J	4.6
Recruitment cost share	X/C	0.021
Productivity	p	1.0

the productivity shock. We fix the AR(1) autoregression coefficient $\rho = 0.979$ and standard deviation $\sigma = 0.007$, similar to the values used by Coles and Moghaddasi Kelishomi (2018). Table 1 below summarizes the calibration.

Given the calibration, Table 2 tabulates various steady-state shares of interest. Nearly 80 % of vacancies are either surviving or reposted following separations, and hence unrelated to current productivity, whereas only about 20 % of the vacancy stock (new vacancies) is directly linked to current productivity. Vacancies have an average age of 7.1 years and have spent an average of 6.8 years matched with a worker since their original inception. Letting X denote aggregate recruitment expenses, its share relative to output X/C is a moderate value of 2.1 %.[15]

The main reason why unemployment and productivity are not perfectly correlated in our model is that vacancies are a positively valued asset in equilibrium. Because of this, following a separation shock s firms will repost their vacancy independent of the current state of labor productivity, p_t. Similarly, surviving vacancies that failed to hire a worker last period will remain open even if the economy experiences a large negative shock. Thus, only the fraction of new entrants within the stock of vacancies responds to the current state of the economy. As a consequence, the magnitude of the correlation between u_t and p_t depends on both the fraction of new vacancies and the average age of a vacancy since its original inception (since p_t follows a highly persistent process). To illustrate the long-lived nature of vacancies in our economy, as well as the impact of the destruction shock, δ, on the distribution of vacancies we turn to Fig. 3.[16] The left panel depicts the cumulative density of vacancies by their age since birth. In our baseline calibration, most vacancies were originally opened more than 3.5 years ago and over 20 % of vacancies are older than 10 years. This indicates that indeed many vacancies in the current pool are the result of a decision to enter the labor market which firms made long ago. The panel also depicts the cumulative density for a destruction rate $\delta = 0.034$. In this scenario *all* firm-worker separations are the result of job destruction. Consequently, the dynamics of vacancies are very different: more than 80 % are newly created and none are reposted. The right panel shows the cumulative density of the time vacancies have spent filled since their original inception, under the baseline calibration. Of course, vacancies in the current pool are not matched with a worker, but many of these vacancies were reposted following an s shock. Indeed, a typical vacancy is matched with a worker, spends some time as a filled job, is reposted as a vacancy, matches with a worker again, and once again spends time as a filled job, etc. Counting the cumulative length of all such spells during which the vacancy was matched with a worker in the past yields the density in the right panel of Fig. 3. About 25 % of vacancies are newly created or surviving and were thus never matched with a worker. The rest have experienced at least one production spell with a median length of cumulative production being about 3.5 years.

3.2. Distributions of key labor market variables and role of non-linearity

Many labor search models potentially feature important nonlinear dynamics (Petrosky-Nadeau and Zhang, 2017). The reason is that the curvature of the matching function induces more pronounced increases in unemployment during recessions than declines

[15] Appendix A.4 characterizes overall recruitment expenses under the heterogeneous sunk entry costs.

[16] Appendix D derives the associated densities for both panels.

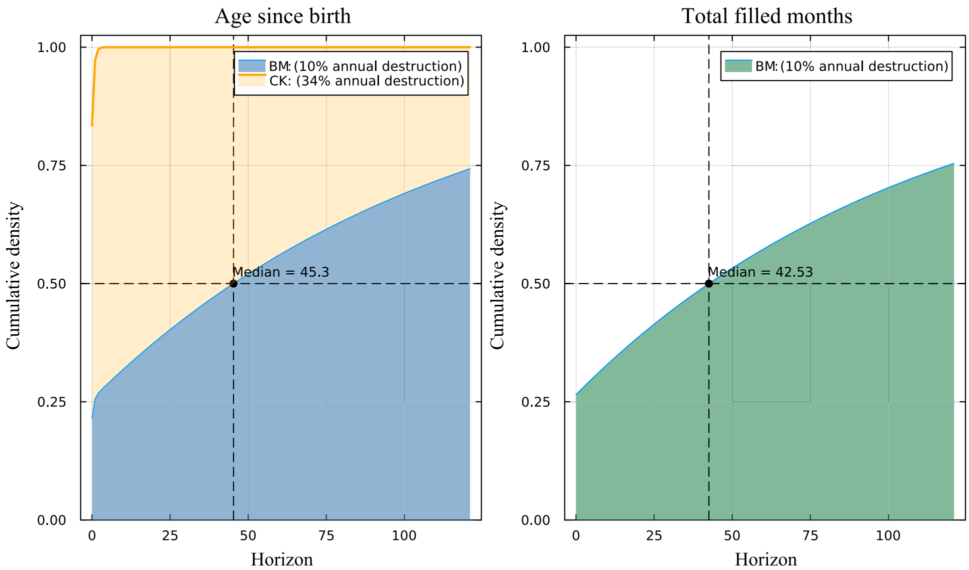

Fig. 3. Cumulative distributions of vacancy age since birth and total time filled. The horizon considered spans 120 months, or 10 years.

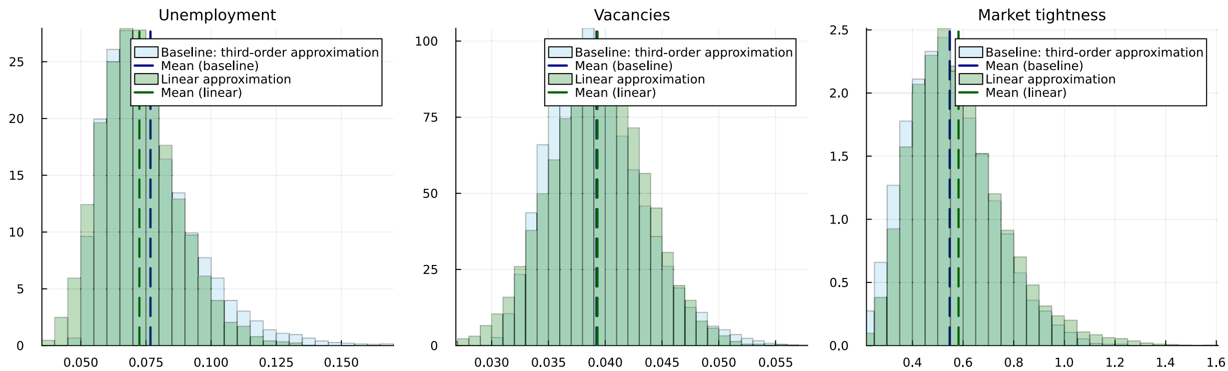

Fig. 4. Histograms of quarterly unemployment, vacancies, and tightness generated from the model solved via a third-order perturbation compared to a first-order approximation. Each series is in levels and consists of 50,000 observations. We truncate observations at the 0.1 % and 99.9 % percentiles. Means of each variant are represented with corresponding vertical lines. The histogram reflects the accumulation of nonlinear effects over many shocks and thus showcases more significant differences.

during expansions. This can be seen via a steady-state approximation. Given the concavity of the matching function, f, unemployment, expressed as a function of the market tightness tightness, $u(\theta) = \tau/[\tau + (1-\delta)f(\theta)]$ is convex. Thus, along the business cycle path, the average rate of unemployment $E[u(\theta)]$ exceeds the steady-state level.

Fig. 4 highlights the importance of non-linearity in our model. All distributions display positive skewness. Using a third-order (our baseline) approximation, the sample means are 7.67 %, 3.92 %, and 0.55 %; a first-order (linear) approximation yields means of 7.24 %, 3.93 %, and 0.58 %. The upward bias in average unemployment relative to its steady state is a consequence of business-cycle asymmetry: downturns tend to be deeper or more persistent than upturns, so the distribution is not symmetric. The third-order approximation, by accounting for these non-linearities, therefore raises the mean unemployment more than the linear approximation. This effect gives rise to a more pronounced right tail for unemployment under the third-order solution; because market tightness moves inversely with unemployment, that amplified right tail translates into greater mass on the left tail of the tightness distribution. In particular, the unemployment distribution has a skewness of 1.35 under a third-order approximation compared with 0.58 under a linear approximation.[17]

[17] Supplemental Appendix A also compares impulse responses between third-order and first-order approximations. In this case, the differences are negligible. We have additionally solved the model via a global numerical procedure, which involves approximating the policy functions with quadratic polynomials and iterating on the Euler equations until finding a fixed point of the coefficients. Supplemental Appendix B describes the procedure. The results are broadly similar. However, we find the results of this method to be rather sensitive to the choice of bounds on the state space as one varies key parameters, namely δ and the vacancy value elasticity ξ. Accordingly, we opt to conduct all the analysis using higher-order perturbation methods.

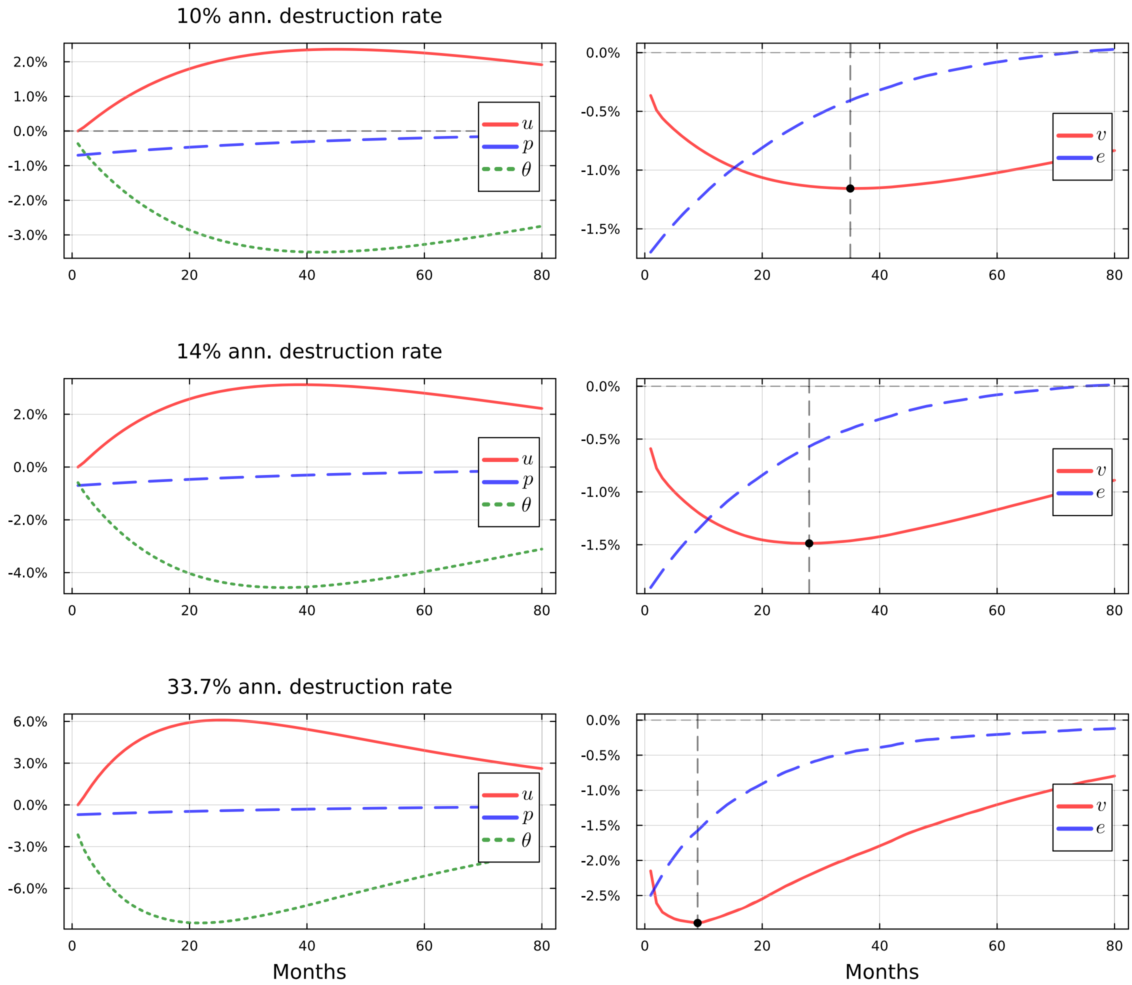

Fig. 5. Impulse response functions to a one standard deviation negative technology shock. The first row corresponds to the benchmark calibration with a $10\,\%$ annual product destruction rate. The second and third rows consider $14\,\%$ and $33.7\,\%$ annual destruction rates, respectively. Each path is the average of $1,000$ replications.

With these features in mind, we solve the model and derive our main numerical results using a third-order perturbation around the deterministic steady-state. We also apply pruning: only the first-order terms are used in computing deviations before plugging into the nonlinear terms, which prevents potentially explosive feedback.

3.3. Impulse response functions and mechanism

To highlight the intuition behind our numerical results, we first examine the model's impulse responses to a one-standard-deviation negative technology shock. Because higher-order solutions are state dependent, we compute generalized impulse responses (GIRFs). For each replication we draw a random sequence of shocks, simulate the model forward with the target shock set to zero to obtain a baseline path, and then simulate a counterfactual path in which we add the one-standard-deviation (negative) impulse at $t = 0$. The GIRF for that replication is the difference between the shocked and baseline paths. The reported path is the average over $1,000$ replications, which smooths out the noise.

On impact vacancies, entry, and the market tightness all decrease (Fig. 5, top row). Entry initially responds the most and only slowly recovers in subsequent periods. This sluggish response arises from the finitely elastic vacancy creation: entry costs are higher for subsequent entrants, which smooths entry. Since vacancies are an asset with a positive value, firms do not voluntarily exit the labor market. Instead, they maintain their vacancies, so the only change in the pool comes from the reduced entry.[18] Thus, vacancies respond very little on impact and achieve their maximum response only around 3 years after the initial shock. Market tightness and

[18] An additional, smaller, reduction in vacancies occurs after the shock since a lower number of entrants today leads to a lower number of surviving and reposted vacancies in the future. However, the overall impact on the mass of vacancies is small, as can be seen in the GIRFs.

Table 3
Contemporaneous correlations with productivity across destruction-rate specifications.

	Data	Model (indexed by annual destruction rate)				
Variable		10 % (BM)	14 % (BDS)	19.6 % (SS)	23.6 % (FR/MS)	33.7 % (CM)
Panel A: Corr(X, p)						
u	−0.43	−0.43	−0.50	−0.57	−0.61	−0.67
θ	0.44	0.53	0.62	0.70	0.74	0.81
v	0.43	0.69	0.78	0.87	0.91	0.95
Panel B: Corr(u, v)						
	−0.91	−0.88	−0.85	−0.80	−0.77	−0.70

Notes: Correlations are contemporaneous. Model moments come from a long monthly simulation (150,000 months) aggregated to quarters; parameters other than δ are recalibrated for each specification. Productivity p is output per hour (BLS PRS85006163). Observables span 1951M1–2003M12. All series are log HP-filtered with $\lambda = 10^5$. Percentages in the column headers denote the target destruction share; values in parentheses report the implied monthly destruction rate δ.

unemployment follow a similar pattern because their behavior is a direct consequence of firm's entry decisions. As highlighted by Fujita and Ramey (2007), this kind of sluggish adjustment is absent in the standard DMP model with infinitely elastic vacancy creation and is consistent with evidence from an identified VAR. Christiano et al. (2016) also obtain a hump-shaped response of vacancies conditional to neutral productivity shocks in a larger structural VAR. The identification approach of the latter is especially persuasive because it considers information on the co-movement of unemployment, vacancies, and labor productivity, inflation, and the relative price of investment.

This sluggishness is determined to a large extend by the value of the destruction shock. When δ is chose to match either 14 % annual destruction rate (middle row) or to $\delta = 3.37\,\%$, which corresponds to the calibration in Coles and Moghaddasi Kelishomi (2018) (bottom row) vacancies reach their peak response much sooner: about 2 or 1 years after the shock.[19] Intuitively, a lower destruction rate implies that most vacancies in the pool are surviving business opportunities from previous periods. As a result, new entrants comprise a small fraction of the pool. Since this is the portion of vacancies that responds to aggregate conditions the reduction of vacancies is small on impact. At the same time both the flows in (entry and separations) and out of vacancies (matching and destruction) are a relatively small fraction of the overall pool, which contributes to a sluggish response of vacancies. Because the behavior of the market tightness and unemployment are directly determined by the behavior of vacancies their response is qualitatively the same.

When vacancies respond sluggishly to shocks, the labor-market variables comove more strongly with past than with current levels of productivity. As a result, the absolute value of the cross-correlation between productivity and the labor market variables is lower. This phenomenon happens for two reasons. First, a sluggish response in vacancies implies lower entry, so a higher fraction of the vacancy pool are vacancies that were created in previous periods (surviving and reposted vacancies). In those previous periods firms were looking at past levels of productivity to make their entry decisions, so those vacancies are correlated to these past productivity values. Second, a slower response in vacancies is associated with lower destruction rates. This means that a greater number of vacancies in the pool were originally opened in prior periods, increasing the average age since inception of vacancies in the vacancy pool. As a result, the correlation between labor market variables and current productivity is weaker when compared with past productivity.

Table 3 highlights this point by presenting the cross-correlation of productivity with vacancies, unemployment, and the market tightness. The second column shows the model-predicted moments in the benchmark calibration (BM) and the third column reports the moments when the destruction shock is calibrated to the 14 % aggregate separation rate from the BDS database. Column 4, referred to as SS, sets $\delta = 0.018$, which matches the calibration in Shao and Silos (2013). Column 5 (FR/MS) matches the destruction rates used in Mercan and Schoefer (2020) and Fujita and Ramey (2007).[20] The final column, referred to as CM, follows Coles and Moghaddasi Kelishomi (2018) in calibrating the destruction rate to account for all separations.

The benchmark calibration in column 2 is able to reproduce the mild cross-correlation of the labor market variables with productivity reasonably well. For vacancies and the market tightness the correlation is slightly higher than that in the data, whereas for unemployment it is at its empirical value. The correlation between vacancies and productivity is somewhat high at 0.69, yet still significantly lower than what the DMP model predicts and the values implied by models in the existing literature. In particular, for the last three columns vacancies are strongly pro-cyclical. A similar pattern emerges for unemployment and market tightness: when we calibrate the destruction shock to values from the existing literature, the cross-correlation is much higher than that in the data. Of course, the correlation does not approach unity even in the case when all separations are due to job destruction (column CM), as the model still features the vacancy smoothing mechanism from Fujita and Ramey (2007). It is worth noting that, for low δ, the model is able to reproduce the mild cross-correlation of the labor variables with productivity while maintaining a strong Beveridge curve. Specifically, in the benchmark calibration Corr(u, v) = −0.88 and, for 14 % annual destruction, Corr(u, v) = −0.85.

[19] For numerical reasons, we set δ to account for 99 % of aggregate separations.

[20] To be precise, Fujita and Ramey (2007) calibrate $\delta = 0.021$ but this small difference does not change the simulated moments in a meaningful way, so we group the two calibrations together.

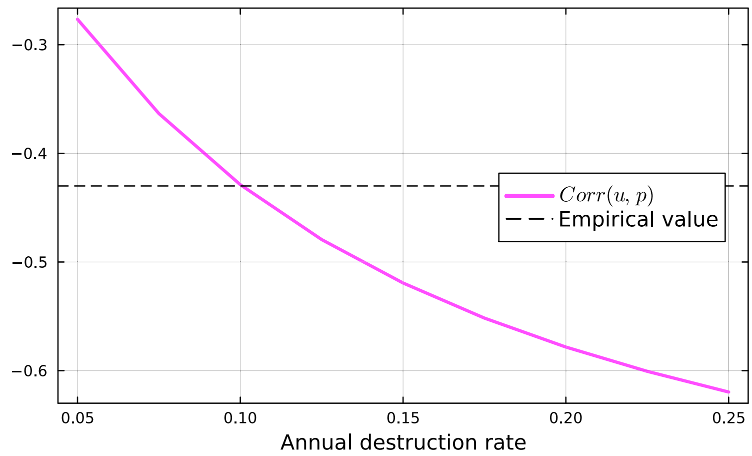

Fig. 6. Correlations between unemployment and labor productivity quarterly averages of simulated data given annualized destruction rates between 5 % and 25 %. See notes in Table 3.

Fig. 6 expands on Table 3 by plotting the contemporaneous correlation between unemployment and labor productivity for an annualized destruction rates ranging from 5 % to 25 %. As the calibrated destruction rate increases, the rise in the magnitude of the correlation is smooth and gradual, and the correlation reaches −0.6 at 22.5 % annual destruction rate. Overall, the figure highlights that our main numerical results are robust to a sensible range of calibrated destruction shocks, δ.

The main mechanism in our model, which breaks the near-perfect correlation between unemployment and productivity, also affects the volatility of unemployment. Unsurprisingly, a higher calibrated δ leads to higher volatility of unemployment. This is the case because higher destruction rates imply that the pool of vacancies is more responsive to productivity shocks. This, in turn, leads to higher volatility in the job-finding rate and ultimately unemployment. The main focus of our paper is not on matching unemployment volatility, nevertheless we show that matching the mild cyclicality of unemployment is compatible with generating empirically plausible volatility. Table 4 varies the elasticity parameter over the grid $\{0.25, 0.50, 1.0, 5.0, 10.0, 15.0\}$, re-calibrating the model at each value. For each case, the table reports the standard deviation of unemployment, SD(u), and its correlation with labor productivity, Corr(u, p). This design isolates any trade-off between generating a mild $u - p$ correlation and delivering sufficient volatility in u while holding the rest of the environment fixed. Under the benchmark $\xi = 1$, the model delivers roughly half the empirical standard deviation of unemployment. Raising ξ increases amplification: the model can match the volatility of unemployment without materially altering the magnitude of its correlation with productivity. For example, at $\xi = 5$ we obtain $\mathrm{Corr}(u, p) = -0.56$ and $\mathrm{SD}(u) = 0.23$.

The overall volatility that the model generates under $\xi = 1$ and $\xi = 5$ is still below that in the data, but much better than the volatility predicted by the baseline DMP model (Shimer, 2005). Of course, the literature has extensively examined the reason why this class of models generally under-predicts the relative unemployment volatility $\mathrm{SD}(u)/\mathrm{SD}(p)$. For example, Ljungqvist and Sargent (2017) introduce the concept of the fundamental surplus, which is the marginal product of a filled job net of payments that must be made regardless of the hiring decision. They show that this measure is a sufficient statistic for characterizing the volatility of market tightness and unemployment across a wide variety of models. Mechanisms that reduce the fundamental surplus raise volatility. Two prominent examples are fixed matching costs, discussed by Pissarides (2009), and Hall and Milgrom (2008)'s credible-bargaining framework; both compress the surplus available to finance vacancy creation and thereby amplify fluctuations. Consistent with this perspective, Christiano et al. (2016) estimate a medium-scale model with unemployment using Bayesian impulse-response matching and show that incorporating fixed matching costs and credible bargaining improves the fit to labor-market and macroeconomic dynamics. These ingredients could be layered onto our framework to more tightly match volatility without altering the core mechanism that delivers the mild unemployment-productivity correlation. However, this exercise is beyond the scope of our paper.

Additionally, we set ξ to unity in our benchmark because it is the value used in Fujita and Ramey (2007) and one of the values considered in Coles and Moghaddasi Kelishomi (2018). The literature has considered larger values of ξ as well: Haefke and Reiter (2020) use 15.88 for their benchmark and Qiu (2023) sets $\xi = 10.7$. Under these alternative values the model can generate more than enough amplification in unemployment to address the Shimer puzzle, yet the correlation between u and p remains relatively mild. It should be noted, however, that the literature has also considered smaller values of the elasticity parameter. For example, Coles and Moghaddasi Kelishomi (2018) also consider a value of $\xi = 0.265$ and Potter (2024) calibrates the value to 0.01. These values inform the choice set of Table 4. Unsurprisingly, when ξ is smaller, there is little volatility in unemployment as congestion costs respond much more to changes in the number of entrants. Thus, Table 4 serves two purposes: it shows that (i) one can generate enough amplification in unemployment and at the same time a relatively mild correlation between unemployment and productivity; (ii) the model generates mild correlation between u and p for a large range of values for ξ.

3.4. Dynamic correlations

A more informative way to characterize the mechanism's time-series behavior is to inspect the dynamic correlations of vacancies with productivity, $\mathrm{Corr}(v_t, p_{t-i})$. When the destruction rate is low, the vacancy stock is relatively concentrated in positions created in

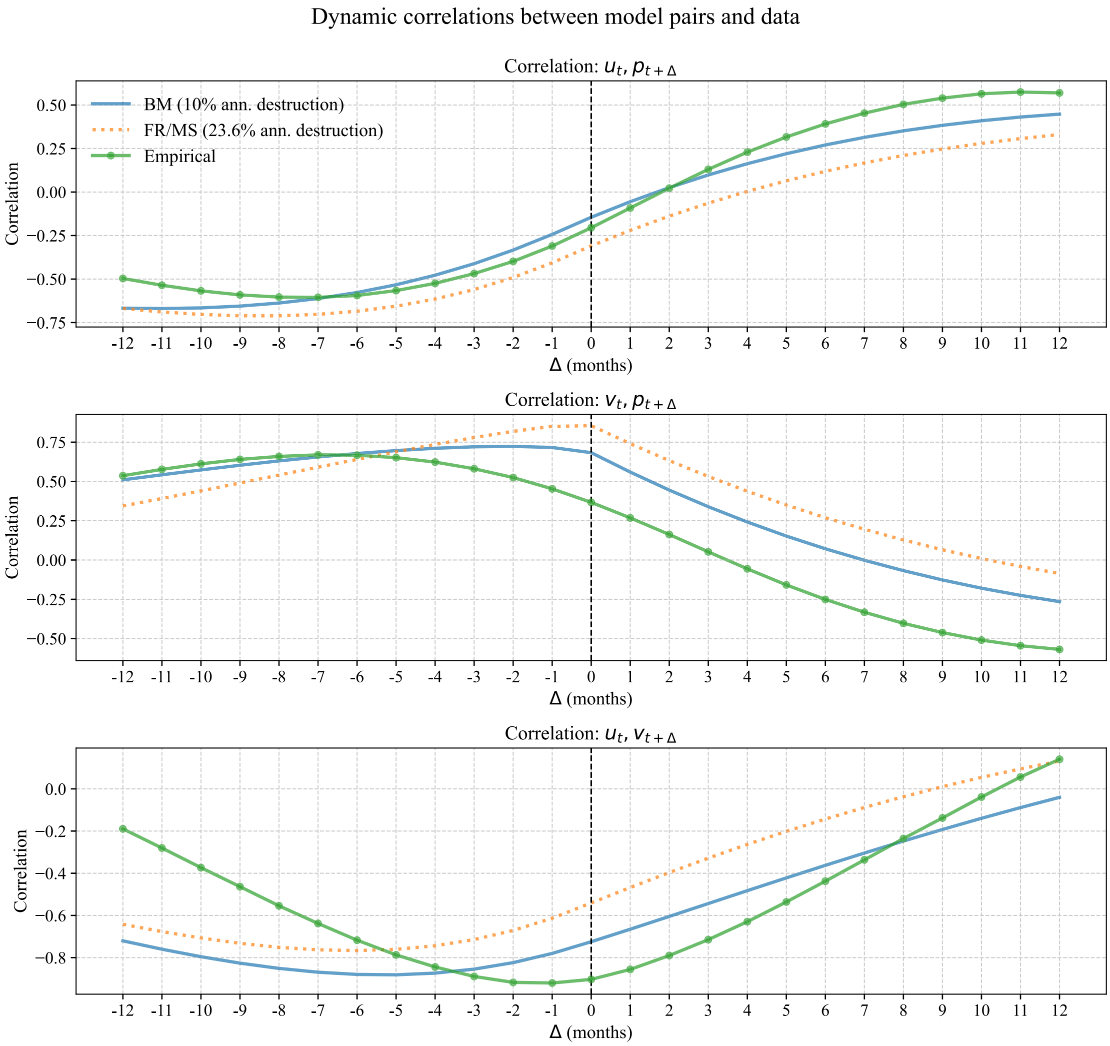

Fig. 7. Dynamic correlations. The horizontal axis in each period depicts the time-shift Δ, measured in months, the vertical axis the correlation coefficient. For each pairwise combination of variables, we consider the data alongside the model calibrated at the benchmark (10 % annual destruction) and the FR/MS specification (23.6 % annual destruction). Correlations are based on the same 150,000 monthly observations used by Table 3. Both the data and model series are logged and HP-filtered with smoothing parameter $\lambda = 10^5$. The data range is 1951M1-2003M12.

Table 4
Summary statistics across values of ξ.

	$\xi = 0.25$	$\xi = 0.50$	$\xi = 1.0$	$\xi = 5.0$	$\xi = 10.0$	$\xi = 15.0$
Corr(u, p)	−0.3	−0.36	−0.43	−0.56	−0.61	−0.636
Std(u)	0.036	0.060	0.095	0.23	0.30	0.355
Std(u)/Std(p)	1.57	2.61	4.1	9.78	13.2	15.4

Moments across different values of the elasticity ξ. The remaining parameters are recalibrated. Moments are based on quarterly averages of 150,000 monthly observations after applying logs and the HP filter with smoothing parameter $\lambda = 10^5$.

earlier periods and therefore remains strongly tied to past productivity. Consistent with this intuition, the benchmark calibration and the 14 % annual destruction calibration exhibit correlations that remain high at short lags and decay only slowly with increasing lags. By contrast, calibrations with larger values of δ (those used in the prior literature) show correlations that fall off much more rapidly. Table 5 reports these results across the different δ values.[21]

[21] We only show the correlations for vacancies, but those for unemployment and the market tightness follow a similar pattern.

Table 5
Dynamic correlations.

Lag	Corr(v_t, p_{t-i}) by destruction rate				
	10 % (BM)	14 % (BDS)	19.6 % (SS)	23.6 % (FR/MS)	33.7 % (CM)
p_{t-1}	0.76	0.83	0.89	0.92	0.92
p_{t-2}	0.79	0.84	0.88	0.88	0.84
p_{t-3}	0.80	0.83	0.84	0.82	0.75
p_{t-4}	0.79	0.81	0.79	0.76	0.65

Each lag is at quarterly frequency. See description for Table 3.

Fig. 7 plots the dynamic correlations, at monthly frequency, for a more granular analysis.[22] For conciseness we focus on the benchmark calibration and the one in which the destruction rate is calibrated as in Fujita and Ramey (2007), FR/MS. The top panel shows the dynamic correlations between unemployment and productivity and the middle one between vacancies and productivity. In both cases the benchmark calibration outperforms. Additionally, the curves corresponding to the two calibrations confirm our results from Table 5: the correlation between vacancies and productivity peaks in the current period and peters off monotonically as lags increase for the FR/MS parameterization, whereas under the benchmark the peak correlation occurs after a several month lag. Importantly, this is the behavior of the data as well: vacancies are more strongly correlated with past levels of productivity than current ones. The third panel shows that the two calibrations produce comparable correlations between unemployment and vacancies. However, they overshoot the magnitude of the comovement between unemployment and lagged vacancies relative to the data.

4. Conclusion

The Diamond-Mortensen-Pissarides framework has been extensively used to analyze labor market dynamics, with important extensions reproducing the relative volatility of unemployment, vacancies, and market tightness. Yet the baseline model cannot generate the empirically observed mild correlation between productivity and labor market variables. We show that including sunk vacancy creation costs and congestion in entry can help reconcile the model with the data and also fit dynamic correlations reasonably well under only a stochastic shock to productivity.

Our model eliminates the near-perfect correlation between the labor market variables and productivity for two reasons. First, congestion in entry induces firms to smooth out entry in response to a shock, which serves to further reduce the correlation between vacancies and productivity. Second, and more importantly, we distinguish carefully between *separation* and *destruction* shocks: whereas separations are the result of a voluntary match dissolution initiated by either the firm or the worker, job destruction captures the loss of business opportunity due to obsolescence, increased competition, etc. Consequently, firms that suffer a separation shock can *repost* their vacancy without having to pay the entry cost again, but firms that suffer a destruction shock lose that ability. In equilibrium, sunk entry costs imply vacancies are a positively-valued asset, so all firms that can repost their vacancy do so. As a result vacancies fall in one of three categories: i) new entrants; ii) surviving vacancies from last period that failed to find a worker; iii) vacancies reposted following a separation shock.

We show that the dynamics of vacancies–and, by extension, unemployment and market tightness–depend on the relative contribution of the destruction shock to total separations. When match dissolutions are primarily driven by destruction shocks, most vacancies stem from new entrants, and the model's dynamics resemble those in the existing literature. Conversely, when separations dominate, most vacancies are surviving or reposted. This raises the average age of vacancies, so their creation is only weakly correlated with current productivity. As a result, the overall correlation of vacancies–and thus unemployment and tightness–with productivity is much lower, and matches the data better than the baseline DMP model. These dynamics also make the impulse response of vacancies and unemployment to productivity shocks more sluggish, consistent with empirical evidence. Finally, calibrating the destruction shock using JOLTS layoff data and recall rates allows the model to match the mild correlation between unemployment and labor productivity observed in the data.

We extend the canonical DMP model only by introducing sunk entry costs and congestion in vacancy creation. We do not explore additional mechanisms, such as match-specific productivity or endogenous separations, which could strengthen or weaken the unemployment-productivity correlation. For example, if separations are endogenous, recessions would bring more separations and stronger correlation, but expansions would reduce destruction and the correlation. We leave thorough investigation of these mechanisms to future research.[23] Other avenues for future research can enhance the understanding of the dynamics between businesses, product lines, and the labor market by explicitly incorporating time-series data and imperfect substitutability between product lines, as suggested by Bilbiie et al. (2012).

[22] We solve the model and simulate artificial data at a monthly frequency, as before, but now also apply the HP filter and compute moments at a monthly frequency.

[23] Although many mechanisms may be important for the magnitude of the cross-correlation, endogenous separations is unlikely to be such a mechanism. Extending our model to i) make the destruction rate a function of productivity, $\delta_t = \bar{\delta} p_t^{-\epsilon}$; ii) to follow the same stochastic process as p_t but with a reasonably scaled variance does not notably alter the magnitude of the correlation. For $\epsilon = 10$, $Corr(u_t, p_t) = -0.36$ and is closer to the baseline for smaller values of ϵ; when $\log(\delta_t) = (1-\rho)\log(\bar{\delta}) + \rho\log(\delta_{t-1}) - \Sigma\epsilon_t$, where Σ is a parameter chosen to match the empirical volatility of separations, $Corr(u_t, p_t) = -0.42$.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.jedc.2025.105205

Appendix A. Derivations

A.1. Job creation condition

First, write the value of a vacancy as

$$Q_t = -\gamma + \beta(1-\delta)\mathbb{E}_t[q(\theta_t)(J_{t+1} - Q_{t+1}) + Q_{t+1}] \tag{A.1}$$

so that

$$\gamma + Q_t - \beta(1-\delta)\mathbb{E}_t Q_{t+1} = \beta(1-\delta)\mathbb{E}_t q(\theta_t)(J_{t+1} - Q_{t+1})$$

Use the variable $K_t = Q_t - \beta(1-\delta)\mathbb{E}_t Q_{t+1}$ and divide by $q(\theta_t)$ to find that

$$\frac{\gamma + K_t}{q(\theta_t)} = \beta(1-\delta)\mathbb{E}_t(J_{t+1} - Q_{t+1}) \tag{A.2}$$

Now, write the value of a filled job as

$$J_t = p_t - w_t + \beta(1-\delta)[J_{t+1} + s(Q_{t+1} - J_{t+1})] \tag{A.3}$$

Subtracting (A.1) from (A.3) we obtain

$$J_t - Q_t = p_t - w_t + \gamma + \beta(1-\delta)\mathbb{E}_t\left[(1 - s - q(\theta_t))(J_{t+1} - Q_{t+1})\right] \tag{A.4}$$

Plug (A.2) into (A.4):

$$\begin{aligned} J_t - Q_t &= p_t - w_t + \gamma + (1 - s - q(\theta_t))\left(\frac{\gamma + K_t}{q(\theta_t)}\right) \\ &= p_t - w_t + \gamma(1-s)\left(\frac{\gamma + K_t}{q(\theta_t)}\right) - (\gamma + K_t) \end{aligned}$$

so that

$$J_t - Q_t = p_t - w_t - K_t + (1-s)\frac{\gamma + K_t}{q(\theta_t)} \tag{A.5}$$

Forward (A.5), plug into (A.2), and rearrange to obtain the Euler equation.

A.2. Wage equation

Let $S_t^W \equiv W_t - U_t$ denote the worker's match surplus and S_t^J denote the firm's match surplus. Nash bargaining implies

$$S^W = \frac{\alpha}{1-\alpha} S_t^J \tag{A.6}$$

From Appendix A, write the firm surplus as

$$S_t^J = p_t - w_t - K_t + (1-s)\frac{\gamma + K_t}{q(\theta_t)}$$

The worker's surplus, in turn, satisfies

$$S_t^W = w_t - b + \beta(1-\delta)(1 - s - f(\theta_t))\mathbb{E}_t S_{t+1}^W$$

Using (A.6) and (2), rewrite the worker's surplus as

$$S_t^W = w_t - b + (1 - s - f(\theta_t))\frac{\alpha}{1-\alpha}\left(\frac{\gamma + K_t}{q(\theta_t)}\right)$$

Using both expressions for the surplus together with the Nash rule (A.6), we have

$$p_t - w_t - K_t + (1-s)\left(\frac{\gamma + K_t}{q(\theta_t)}\right) = \frac{1-\alpha}{\alpha}\left[w_t - b + (1 - s - f(\theta_t))\frac{\alpha}{1-\alpha}\left(\frac{\gamma + K_t}{q(\theta_t)}\right)\right]$$

Algebraically rearranging for the wage yields

$$w_t = \alpha\big(p_t - K_t + \theta_t(\gamma + K_t)\big) + (1-\alpha)b$$

A.3. Steady state

Each steady state condition follows trivially from the general equilibrium conditions except (16):

$$e = \delta(v + 1 - u)$$

To see this, first rearrange the steady-state condition for vacancies as

$$v = \frac{s(1-\delta)(1-u) + e}{\delta + (1-\delta)q(\theta)}$$

Then note that

$$u = \frac{v}{\theta} = \frac{s(1-\delta)(1-u) + e}{\delta\theta + (1-\delta)f(\theta)}$$

Combining this with the steady-state Beveridge curve (15), we have

$$\frac{s(1-\delta)(1-u) + e}{\delta\theta + (1-\delta)f(\theta)} = \frac{\tau}{\tau + (1-\delta)f(\theta)}$$

Rearrange for e to find

$$e = \delta v + (1-u)(\tau + s(1-\delta)) = \delta(v + 1 - u)$$

A.3.1. Further properties of $e(\theta)$ and $K(\theta)$

Recall that we have characterized entrants $e(\theta)$ and the flow entry cost $K(\theta)$ in Equation (A.8) of the main text:

$$e(\theta) = \delta\left(\frac{\theta\tau + (1-\delta)f(\theta)}{\tau + (1-\delta)f(\theta)}\right),\ K = \frac{r+\delta}{1+r}e(\theta)^{1/\xi}x_m$$

As $\delta \to 0$, $e \to 0$. Moreover, as $\theta \to 0$, $q(\theta) \to 1$, so that $e \to \frac{\delta}{1-\delta}(\tau + 1 - \delta)$. As $\theta \to \infty$, $q(\theta) \to 0$, and $e \to \infty$.

Note that, in contrast to γ, K rises with θ. The average hiring cost arising from investment in a product line is $K/q(\theta)$ therefore rises both directly from K and also because of shorter duration of a vacancy.

We can also characterize the steady state equilibrium entirely in terms of market tightness. Combine (12) and (14) to express the steady-state job creation condition as

$$\frac{\gamma + K}{q(\theta)} = \frac{(1-\delta)(1-\alpha)(p-b-K)}{r + \tau + \alpha(1-\delta)f(\theta)}$$

and rearrange as

$$\frac{\gamma + K}{q(\theta)} + \frac{(1-\delta)(1-\alpha)K}{r + \tau + \alpha(1-\delta)f(\theta)} = \frac{(1-\delta)(1-\alpha)(p-b)}{r + \tau + \alpha(1-\delta)f(\theta)}. \tag{A.7}$$

Let e and K be implicit functions of θ as follows:

$$e(\theta) = \delta\left(\frac{\theta\tau + (1-\delta)f(\theta)}{\tau + (1-\delta)f(\theta)}\right),\ K = \frac{r+\delta}{1+r}e(\theta)^{1/\xi}x_m \tag{A.8}$$

Then, given $K(\theta)$, (A.7) characterizes market tightness completely.

$$\frac{\gamma}{q(\theta)} = \frac{(1-\alpha)(p-b) - \gamma\alpha\theta}{r+s}$$

A.3.2. Proof of Proposition 1: Existence and uniqueness

Now, let us re-examine the job creation condition:

$$\frac{\gamma + K(\theta)}{q(\theta)} = (1-\delta)\frac{(1-\alpha)(p-b-K(\theta))}{r + \tau + \alpha(1-\delta)f(\theta)}$$

Note that the left-hand side $g_l(\theta)$ is increasing in θ and the right-hand $g_r(\theta)$ side is decreasing. Thus, equality, if it occurs, can occur only once. Since each side is a continuous function of θ, to show existence it suffices to find θ^* such that $g_l(\theta^*) < g_h(\theta^*)$ and θ^{**} such that $g_l(\theta^{**}) > g_h(\theta^{**})$.

Noting that $K(\theta) \to \infty$ as $\theta \to \infty$, we note that $g_r(\theta) \geq 0$, and we can choose θ^{**} to be the value such that $K(\theta^{**}) = p - b$.

Now choose $\theta^* = 0$, the lowest possible value of θ. Note that $q(0) = 1$, $f(0) = 0$, and $K(0) = 0$. Hence,

We need to show that

$$\gamma < (1-\delta)(1-\alpha)\frac{p-b}{r+\tau} \Leftrightarrow$$

$$\gamma(r+\tau) < (1-\delta)(1-\alpha)(p-b)$$

A.4. Share of recruitment costs to output

Heterogeneous sunk entry costs affect the level of recruiting costs in the economy. Provided $Q < x_m$, then total entry costs X are

$$X \equiv \int_0^Q xg(x)dx = \int_0^Q x x_m^{-\xi} \xi x^{\xi-1} dx = x_m^{-\xi} \frac{\xi}{\xi+1} Q^{\xi+1}$$

and are thus proportional to $Q^{\xi+1}$. Using the free entry condition to substitute for x_m, we have $X = e\frac{\xi}{\xi+1}Q$. The value of a vacancy is thus clearly important for the level of recruiting costs, a first moment, as well as business cycle dynamics. Note that gross output is $Y = (1-u)p$, and net output (GDP) is $C = Y - X$. Thus, X/C represents the share of recruiting costs relative to GDP.

A.5. Elasticity of market tightness with respect to productivity

We examine the steady-state elasticity of market tightness with respect to productivity. First, apply logs to the steady state job creation condition (13):

$$\log\left(\frac{\gamma + K}{q(\theta)}\right) = \log\left(\frac{1-\delta}{r+\tau}\right) + \log(p - w - K)$$

Now differentiate with respect to $\log\theta$, treating w and K as constant. We obtain

$$-\frac{1}{\frac{\gamma}{q(\theta)}} \frac{\gamma + K}{q(\theta)^2} \frac{\partial q}{\partial \log\theta} \epsilon_{\theta,p} = \frac{1}{p - w - K}$$
$$\frac{1}{\frac{\gamma}{q(\theta)}} \frac{\gamma + K}{q(\theta)} \eta_L \epsilon_{\theta,p} = \frac{1}{p - w - K}$$

Rearrange as

$$\epsilon_{\theta,p} = \frac{1}{\eta_L(p - w - K)}$$

compared to $(1/(\eta_L(p-w)))$ in the baseline model. The direct effect of K is thus to increase amplification. However, the congestion channel dampens amplification. The congestion channel dissipates with higher ξ, disappearing as $\xi \to \infty$.

Appendix B. Calibration

Our baseline calibration assumes the following targets: $\eta_{w,z} = 0.6, r^{ann} = 0.04, p = 1, b = 0.9, \tau = 0.034, \delta^{ann} = 0.10, \xi = 1, \overline{f} = 0.45, \overline{q} = 0.80, \gamma = 0$. Now, proceed as follows:

1. Separation rates and rates of time preference. Calculate

$$\delta = 1 - (1 - \delta^{ann})^{1/12}; \qquad s = (\tau - \delta)/(1 - \delta); \qquad \rho = (1 + r^{ann})^{1/12} - 1$$

2. Matching rates, tightness, unemployment, and vacancies. Calculate

$$f = \overline{f}/(1-\delta) \qquad q = \overline{q}/(1-\delta) \qquad \theta = f/q$$
$$u = \tau/(\tau + (1-\delta)f) \qquad v = \theta u$$

3. New entrants $e = \delta(v + 1 - u)$
4. Level parameter ν_L of matching function. Use root finding method to calculate ν_L that solves $f = (1 + \theta^{-\nu_L})^{-1/\nu_L}$
5. Calibration of z and α, solving for K and w as intermediate steps.
   - Define $sr = (r + \tau)/((1-\delta)q)$ and note that $sr = (p - w - K)/K$. Recall that $\alpha = \eta_{w,z} w/p$.
   - Plug these expressions into the wage equation

$$w = \alpha(p + (\theta - 1))K) + (1 - \alpha)b$$
$$p - (sr + 1)K = \frac{\eta_{w,z} w}{p}(p + (\theta - 1)K) + (1 - \frac{\eta_{w,z} w}{z})b$$

   - This characterizes a quadratic equation in terms of K. The solution can be characterized as follows. Given

$$a = (1 - \eta_{w,z})p + \eta_{w,z} b; \qquad A = \eta_{w,z}(\theta - 1)(sr + 1);$$
$$B = a(sr + 1) + \eta_{w,z}(\theta - 1)p; \qquad C = p(1 - \eta_{w,z})(p - b);$$

   and the discriminant $D = B^2 - 4AC$ we have $K = \frac{B - \sqrt{D}}{2A}$, where we verify that this root uniquely implies a positive value of K.
   - Now calculate

$$w = p - (sr + 1)K$$
$$\alpha = (w - b)/(p - K + \theta K - b)$$

6. Calibration of x_m. Finally, calculate

$$Q = \frac{1+r}{r+\delta}K$$
$$x_m = \frac{Q}{e^{1/\xi}}$$

Appendix C. Alternate parameterization

We consider an equivalent microfoundation of the entry friction. Assume a firm can develop a product line at sunk entry cost ke_t^{ϕ}, so that the entry cost increases in the number of entrants. The value of a vacant firm with a product line is thus $Q_t = ke_t^{\phi}$. The flow entry cost becomes

$$K_t = k\mathbb{E}_t\left(e_t^{\phi} - \beta(1-\delta)e_{t+1}^{\phi}\right)$$

The mapping between (x_m, ξ) in the original parameterization and (k, ϕ) is $k = x_m$ and $\phi = 1/\xi$. The DMP model arises by letting $k \to 0$ and $\phi \to 0$.

Appendix D. Deriving the distribution of vacancies by age and cumulative production spells

We derive the steady-state cumulative distributions of vacancies by age and time filled visualized in Fig. 3. We first focus on age. Jobs are either open (O) or filled (F). The probability that a job is open, conditional on it being alive, is $Pr(O|Alive) \equiv \Pi_0 = \frac{s}{s+q(\theta)}$. Thus, the probability that a job is filled, conditional on it being alive, $Pr(F|Alive) \equiv \Pi_F = \frac{q(\theta)}{s+q(\theta)}$.

Let $P_a \equiv Pr(\text{O at age a}|Alive)$ denote that the probability that a job is open at a particular age (in months) conditional on it being alive. Then we have the law of motion

$$\begin{aligned} P_a &= (1-q(\theta))P_{a,-1} + s(1-P_{a,-1}) \\ &= s + \lambda P_{a,-1} \end{aligned}$$

where $\lambda = 1 - s - q(\theta)$. Now, calculate the difference

$$\begin{aligned} P_a - \Pi_0 &= \lambda P_{a,-1} - s\frac{1-s-q(\theta)}{s+q(\theta)} \\ &= \lambda(P_{a,-1} - \Pi_0) \end{aligned}$$

Note that a vacancy is initially open when alive $P_0 = 1$. Hence, we can iterate the law of motion as

$$P_a - \Pi_0 = \lambda^a(1-\Pi_0)$$

Now, write

$$P_a = \Pi_0 + \lambda^a(1-\Pi_0) = \frac{s}{s+q(\theta)} + \lambda^a\frac{q(\theta)}{s+q(\theta)}$$

Next, we calculate the probability that a vacancy is open at age a. Note that $Pr(\text{O at age a}) = (1-\delta)^a P_a$. So,

$$Pr(\text{O at age a}) = (1-\delta)^a\left[\frac{s}{s+q(\theta)} + \lambda^a\frac{q(\theta)}{s+q(\theta)}\right]$$

Let P_a^v denote the fraction of vacancies open now and born a periods ago. Notice that, at birth, this fraction is the ratio of entrants to vacancies e/v. Thus

$$P_a^v = \frac{e}{v}Pr(\text{O at age a})$$

Writing $e/v = \delta[1 + (1-\delta)q(\theta)/\tau]$, we have

$$P_a^v = \delta[1 + \frac{(1-\delta)q(\theta)}{\tau}](1-\delta)^a\left[\frac{s}{s+q(\theta)} + \lambda^a\frac{q(\theta)}{s+q(\theta)}\right]$$

Next, we examine the distribution of vacancies by cumulative filled spells. Define

$$\begin{aligned} P_t^O(a) &= Pr(state = O, \text{filled time} = a|\text{alive at } t) \\ P_t^F(a) &= Pr(state = F, \text{filled time} = a|\text{alive at } t) \end{aligned}$$

The laws of motion are

$$\begin{aligned} P_{t+1}^O(a) &= (1-q)P_t^O(a) + sP_t^F(a-1) \\ P_{t+1}^O(a) &= qP_t^O(a) + (1-s)Q_t(a-1) \end{aligned}$$

Let $Mass(a) = e\sum_{t=0}^{\infty}(1-\delta)^t P_t^O(a)$. Thus, the density of vacancies by cumulative open spell is $g(a) = Mass(a)/v$.

Define the probability generating functions

$$\tilde{P}_t^O(z) = \sum_a \tilde{P}_t^O(a) z^a$$
$$\tilde{P}_t^F(z) = \sum_a \tilde{P}_t^F(a) z^a$$

The matrix representation of the law of motion is

$$\begin{bmatrix} \tilde{P}_t^O(z) \\ \tilde{P}_t^F(z) \end{bmatrix} = \begin{bmatrix} 1-q & sz \\ q & (1-s)z \end{bmatrix} \begin{bmatrix} \tilde{P}_{t-1}^O(z) \\ \tilde{P}_{t-1}^F(z) \end{bmatrix}$$

Let $A(z)$ denote the transition matrix. Then, iterating on the law of motion and using the fact that $\tilde{P}_0^O(z) = 1$ and $\tilde{P}_0^F(z) = 0$ yields

$$\begin{bmatrix} \tilde{P}_t^O(z) \\ \tilde{P}_t^F(z) \end{bmatrix} = A(z)^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

To obtain the first element, we pre-multiply by $[1 \quad 0]$ as

$$\tilde{P}_t^O(z) = \begin{bmatrix} 1 & 0 \end{bmatrix} A(z)^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Let $S(z) = \sum_{a \geq 0} Mass(a) z^a = e \sum_{t=0}^{\infty} (1-\delta)^t \tilde{P}_t^O(z)$.

Note that $v = S(1)$. Thus, the cumulative distribution function has the form $G(z) = S(z)/S(1)$. Thus,

$$\begin{aligned} S(z) &= e \sum_{t=0}^{\infty} (1-\delta)^t \begin{bmatrix} 1 & 0 \end{bmatrix} A(z)^t \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= e \begin{bmatrix} 1 & 0 \end{bmatrix} \sum_{t=0}^{\infty} [(1-\delta) A(z)]^t \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{aligned}$$

Note that

$$B \equiv I - (1-\delta) A(z) = \begin{pmatrix} \delta + (1-\delta) q & -(1-\delta) sz \\ -(1-\delta) q & 1 - (1-\delta)(1-s) z \end{pmatrix}$$

Thus,

$$B^{-1} = \frac{1}{Det(B)} \begin{bmatrix} 1 - (1-\delta)(1-s) z & (1-\delta) sz \\ (1-\delta) q & \delta + (1-\delta) q \end{bmatrix}$$

We next, let $A_0 \equiv \delta + (1-\delta) q$ and evaluate $Det(B)$:

$$\begin{aligned} Det(B) &= A_0 [1 - (1-\delta)(1-s) z] - [-(1-\delta) sz][-(1-\delta) q] \\ &= A_0 [1 - (1-\tau) z] - (1-\delta)^2 sqz \\ &= A_0 - [A_0 (1-\tau) + (1-\delta)^2 sq] z \end{aligned}$$

Hence, the $(1,1)$ entry of B^{-1} is

$$\frac{1 - (1-\tau) z}{A_0 - [A_0 (1-\tau) + (1-\delta)^2 sq] z}$$

Thus,

$$S(z) = e \frac{1 - (1-\tau) z}{A_0 - [A_0 (1-\tau) + (1-\delta)^2 sq] z}$$

We evaluate

$$S(1) = e \frac{\tau}{A_0 \left(\tau - \frac{(1-\delta)^2 sq}{A_0} \right)}$$

Let $b_0 \equiv 1 - \tau + (1-\delta)^2 sq / A_0$, so that $S(1) = e\tau / (A_0 (1 - b_0))$. Thus, $G(z)$ satisfies

$$G(z) = \frac{[(1 - (1-\tau) z)] A_0 (1 - b_0)}{A_0 (1 - b_0 z) \tau}$$

Canceling and simplifying,

$$G(z) = \frac{[(1 - (1-\tau) z)](1 - b_0)}{(1 - b_0 z) \tau}$$

and thus noting the geometric series expansion:

$$G(z) = \frac{1 - b_0}{\tau} [1 - (1-\tau) z] \sum_{k \geq 0} (b_0 z)^k$$

$$= \frac{1-b_0}{\tau}\left[\sum_{k\geq 0}(b_0 z)^k - (1-\tau)z\sum_{k\geq 0}(b_0 z)^k\right]$$

Expand powers and transform indices:

$$G(z) = \frac{1-b_0}{\tau}\left[\sum_{a\geq 0}(b_0)^a z^a - (1-\tau)\sum_{a\geq 1}(b_0)^{a-1} z^a\right]$$

From this expression, the initial probability mass satisfies $g(0) = (1-b_0)/\tau$. Hence, the general expression for the probability mass function is

$$g(a) = \frac{1-b_0}{\tau}[b_0^a - (1-\tau)b^{a-1}]$$

given $a \geq 1$.

References

Barnichon, R., 2010. Productivity and unemployment over the business cycle. J. Monet. Econ. 57 (8), 1013–1025.
Beaudry, P., Green, D.A., Sand, B.M., 2018. In search of labor demand. Am. Econ. Rev. 108 (9), 2714–2757.
Bernard, A.B., Redding, S.J., Schott, P.K., 2010. Multiple-product firms and product switching. Am. Econ. Rev. 100 (1), 70–97.
Bilbiie, F.O., Ghironi, F., Melitz, M.J., 2012. Endogenous entry, product variety, and business cycles. J. Polit. Econ. 120 (2).
Blanchard, O.J., Summers, L.H., 1986. Hysteresis and the european unemployment problem. NBER Macroecon. Annu. 1, 15–78.
Broda, C., Weinstein, D.E., 2010. Product creation and destruction: evidence and price implications. Am. Econ. Rev. 100 (3), 691–723.
Broer, T., Druedahl, J., Harmenberg, K., Öberg, E., 2025. The unemployment-risk channel in business-cycle fluctuations. Int. Econ. Rev.
Christiano, L.J., Eichenbaum, M.S., Trabandt, M., 2016. Unemployment and business cycles. Econometrica 84 (4), 1523–1569.
Coles, M.G., Moghaddasi Kelishomi, A., 2018. Do job destruction shocks matter in the theory of unemployment? Am. Econ. J. 10 (3), 118–136.
Comin, D., Gertler, M., 2006. Medium-term business cycles. Am. Econ. Rev. 96 (3), 523.
Den Haan, W.J., Ramey, G., Watson, J., 2000. Job destruction and propagation of shocks. Am. Econ. Rev. 90 (3), 482–498.
Diamond, P.A., 1982. Aggregate demand management in search equilibrium. J. Polit. Econ. 90 (5), 881–894.
Faberman, R.J., et al., 2004. Gross Job Flows over the Past Two Business Cycles: Not all 'Recoveries' are Created Equal. Technical Report.
Fujita, S., Moscarini, G., 2017. Recall and unemployment. Am. Econ. Rev. 107 (12), 3875–3916.
Fujita, S., Ramey, G., 2007. Job matching and propagation. J. Econ. Dyn. Contr. 31 (11), 3671–3698.
Gabrovski, M., 2019. Simultaneous innovation and the cyclicality of r&d. Rev. Econ. Dyn. forthcoming .
Gabrovski, M., 2022. Coordination frictions and economic growth. Macroecon. Dyn. forthcoming.
Gervais, M., Jaimovich, N., Siu, H.E., Yedid-Levi, Y., 2015. Technological learning and labor market dynamics. Int. Econ. Rev. 56 (1), 27–53.
Haefke, C., Reiter, M., 2020. Long live the vacancy. No. 13666. IZA Discussion Papers.
Hagedorn, M., Manovskii, I., 2008. The cyclical behavior of equilibrium unemployment and vacancies revisited. Am. Econ. Rev. 98 (4), 1692–1706.
Hagedorn, M., Manovskii, I., 2011. Productivity and the labor market: comovement over the business cycle. Int. Econ. Rev. 52 (3), 603–619.
Hall, R.E., Milgrom, P.R., 2008. The limited influence of unemployment on the wage bargain. Am. Econ. Rev. 98 (4), 1653–1674.
Lam, J.F., Qiu, X., 2022. Job Recalls and Worker Flows Over the Life Cycle. Technical Report. Technical report, University of Pennsylvania.
Lee, Y., Mukoyama, T., 2015. Entry and exit of manufacturing plants over the business cycle. Eur. Econ. Rev. 77, 20–27.
Ljungqvist, L., Sargent, T.J., 2017. The fundamental surplus. Am. Econ. Rev. 107 (9), 2630–2665.
Mercan, Y., Schoefer, B., 2020. Jobs and matches: quits, replacement hiring, and vacancy chains. Am. Econ. Rev. 2 (1), 101–124.
Mortensen, D.T., Nagypal, E., 2007. More on unemployment and vacancy fluctuations. Rev. Econ. Dyn. 10 (3), 327–347.
Petrosky-Nadeau, N., Zhang, L., 2017. Solving the DMP model accurately. Quant. Econom. 8, 611-650
Pissarides, C.A., 2000. Equilibrium Unemployment Theory. MIT press.
Pissarides, C.A., 2009. The unemployment volatility puzzle: is wage stickiness the answer? Econometrica 77 (5), 1339–1369.
Potter, T., 2024. Destabilizing search technology. J. Monet. Econ. 145, 103557.
Qiu, X., 2023. Vacant Jobs. University of Pennsylvania .
Shao, E., Silos, P., 2013. Entry costs and labor market dynamics. Eur. Econ. Rev. 63, 243–255.
Shimer, R., 2005. The cyclical behavior of equilibrium unemployment and vacancies. Am. Econ. Rev. 95 (1), 25–49.